A Dragon-based Parenting Model

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Abstract: Power On account of original novel, A Song of Ice and Fire, we propose an enforceable parenting model for the three firedragons in this paper. Considering the three influential factors, it quantifies the value of them. Using small organisms to substitute dragons' growth curve and required living area approximately, and realizes the virtual condition. We first analogy the three dragons to the most similar animals in the natural world in some aspects, such as birds since the huge similarities in muscle density, appearance, energy consumption, and consuming flight energy. In addition, they are endowed with some other features, such as growing indefinitely, the ability to fire, and extremely low death probability. In order to analyses this virtual object reasonably, the model we build is based on the existing statistic of birds, and we improved it from setting its physique size which is affected by food and environment. Additionally, they can migrate to different areas of residence as a result of changing from seasons and climates. Aiming to raise this virtual animal, this article applies many analyses in serval aspects from existing similar creatures. We comprehensively analyses it based on the Final Value model and the Survival Area model. For instance, we consider the basic metabolic energy consumption, the smoldering energy consumption, the consuming flight, the probability of death, and the combat value. Later, it combined the Analytic Hierarchy Process to divide the three influence factors into different weights, and get the estimate value finally. Moreover, the habitable space of the dragons is measured in the Survival Area Model. To get reliable data, we surveyed substantial papers and literatures. The use of objective and reasonable data to determine the size of the living area, and put forward the problem of value in the long term.

1. Introduction

The image of the Three Dragons is proposed by Daenerys Targaryen, the "Mother of the Dragons" in the fictional television series "Game of Thrones" adapted from *A Song of Ice and Fire,* which is shown in Figure1 [1]. Their hatchling weighs just 10 kilograms and grows to around 30 to 40 kilograms a year later. According to the novel, dragons feed on meat, have unlimited life and grow indefinitely throughout their lives. Their final size depends on external factors such as the quantity of food and environment [2]. Although a few descriptions of the characteristics of the three dragons have been described in the original book, the characteristics of the dragon, behavioral habits and its interaction with the environment have not been clearly demonstrated. Bringing the fictional dragon to reality, it is assumed that the basic biology of dragons described in the book is all in line with the reality [3]. Then there will be a series of practical problems need to be solved such as the ecological impact and requirements of the dragon, the energy consumption and calorie intake requirements, the area needed to support the survival of the three dragons and the migratory activities of the dragon…

Figure1. Dragons

2. Final Value Models

2.1 Energy Consumption Model

In order to calculate the energy required for the dragon's survival, we made a model of energy consumption. In the model of energy consumption, we divide the energy consumption of the dragon into three parts, that is, the basic energy expenditure necessary for basic life activities such as breathing, the energy consumed by the dragon when breathing fire and the energy consumed by the dragon when flying. We made a series of assumptions during the making of the model, and the derivation process will be shown later. The meanings of all the symbols involved in the formula are also given below. The whole formula is as follows:

$$E_{\text{total}} = E_{\text{b}} + \mathrm{E}_{\text{fire}} + E_{\text{fly}} \tag{1}$$

$$E_{\text{total}} = [\alpha(\mathrm{mg})^{\frac{3}{4}} + (\mathrm{T_b} - \mathrm{T_a})\mathrm{C}]t_1 + \mathrm{n}\pi(k_1\mathrm{r})^2\mathrm{h}\rho\frac{1}{\mathrm{M_{mol}}}\Delta\mathrm{H} + [50.7\mathrm{m}^{0.72}]t_2 \tag{2}$$

2.1.1 Basic Energy Expenditure

The first component of the energy consumption model is the calculation of basic energy expenditure (BEE). By assuming that the energy consumption principle of dragons is similar to that of other animals, we use McMahon's structural theory and Scholander - Irving (s-i) model to estimate the basic energy consumption of dragons.

$$E_{\text{b}} = [\alpha(\mathrm{mg})^{\frac{3}{4}} + (\mathrm{T_b} - \mathrm{T_a})\mathrm{C}]t_1 \tag{3}$$

Where:

E_b is basic energy expenditure (BEE); α is a parameter in McMahon's structural theory;

m is the mass of the dragon; g is the acceleration of gravity;

T_a is the external temperature; T_b is the body surface temperature of the dragon;

C is a coefficient of Scholander - Irving model; t_1 is the survival time

Through this formula, we can estimate the basic energy consumption of the dragon based on the dragon's mass m and climatic conditions T_a. It should be noted that the temperature used here is also an important indicator for us to consider the impact of different climates on the energy consumption of dragons in section seven.

2.1.2 Breathing Fire Energy Expenditure

After the calculation of the basic energy expenditure (BEE), the next goal is to calculate the amount of heat consumed by a dragon breathing fire. We hypothesized that the dragon sprayed the fire by spraying ether, and simply substituted the heat required by the volume of the fire ejected by the dragon. Then used the concept of heat of combustion to calculate.

$$\mathrm{E}_{\text{fire}} = \mathrm{n}\pi(k_1\mathrm{r})^2\mathrm{h}\rho\frac{1}{\mathrm{M_{mol}}}\Delta\mathrm{H} \tag{4}$$

Where:

E_{fire} is the energy expended while breathing fire; k_1 is the proportional coefficient

m is the mass of the dragon; h is the height of the flame

ρ is the density of the ether; M_{mol} is the molar mass

ΔH is the heat of combustion; n is the number of fires

This formula is derived from the calculation of the heat of combustion. By this formula, we can calculate the energy that the dragon needs to expand when breathing fire based only on the mass m of the dragon and the height h of the flame the dragon spewing.

2.1.3 Flight Energy Expenditure

The last formula for calculating energy consumption is related to the flight of dragons. We use the energy consumption of birds to make an approximate substitution for the energy consumption of dragons in flight. The Least-squares regression analysis was used to fit the data of many different birds to obtain the following formula.

$$E_{\text{fly}} = [50.7\text{m}^{0.72}]\ t_2 \qquad (5)$$

Where:

E_{fly} is the energy consumed in flight; m is the mass of the dragon; T_2 is the flight time

By using this formula, we can calculate the energy consumption of a dragon in flight directly from its mass m and flight time t_2.

2.2 Risk Probability Model

According to the novel *A Song of Ice and Fire*, the dragon has unlimited life and can withstand huge damage. However considering the eyes of the dragon is its weakness, we decided to use the Poisson Distribution to estimate the probability of the death of the dragon.

$$Risk = \sum_{k_2=1}^{n_1} \frac{\lambda^{k_2}}{k_2!} e^{-\lambda} \qquad (6)$$

Where:

d is the number of dead dragons;

λ is the average number of random events per unit time (or per unit area);

n_1 is the total number of dragons; n_2 is the number of dead dragons

After an estimate of λ, we can use the Poisson distribution to calculate the probability of dragon death, setting the stage for a subsequent evaluation of the value of the dragon.

2.3 Fight Value Model

By putting the dragon in a realistic environment, we made a quantitative analysis of its value and finally decided to analyze its value from the perspective of war and politics. We assume that the dragon fights through fire, and further estimate the combat value of the dragon by using the formula of energy consumption during the fire.

$$Fight\ value = \text{n}\pi(k_3\text{m})^2 hbc_p \qquad (7)$$

Where:

k_3 is the proportional coefficient; m is the mass of the dragon;

h is the height of the flame; b is the number of people per unit area;

c_p is the value of people; n is the number of fires

Through this formula, we can directly calculate its value through the volume of the dragon's flame, which is helpful for the calculation of the final value.

2.4 Characteristics of The Scale

From previous illustrations, we are able to integrate all these features together.

In order to unify the variables or feature range in the data and avoid the scope gap between variables in the original data exceeding that of the final model being determined by only one of the models, we used the feature scaling method to adjust the range before using the data. The formula is as follows:

$$x' = \frac{x - \min(x)}{\max(x) - \min(x)} \qquad (8)$$

$$Z = \frac{\text{x} - \bar{x}}{S_x} \qquad (9)$$

Where:

x is an original value; x' is the adjusted value

max(x) is the largest possible value of x; min(x) is the smallest possible value of x

Z is the adjusted value; $\overline{x}$ is average number of x

S_x is the standard deviation

For E_{total}, we use the first feature scaling method. Given that the range of Rsik is already [0,1], we are not going to adjust its range. For *Fight value*, we think Z score is a more appropriate feature scaling method.

After the feature scaling, E_{total}, Risk, Fight value become E′total, Risk′, *Fight value′*. As we can see, E'_{total} and $Risk'$ have an inverse linear relationship with the final value, therefore, we use the modified scaling method to correct it for positive correlation.

$$E'_{total} = \frac{E_{total}(10) - E_{total}}{E_{total}(M) - E_{total}(10)} \tag{10}$$

$$Risk' = -\ Risk \tag{11}$$

$$Fight\ value' = \frac{Fight\ value - \overline{Fight\ value}}{S_{x(Fight\ value)}} \tag{12}$$

$$s.t.\begin{cases} T_b < 43℃ \\ T_a < 60℃ \\ n < \frac{E_s - E_b - E_{fly}}{\pi(kr)^2 h\rho \frac{1}{M_{mol}} \Delta H} \end{cases} \tag{13}$$

2.5 Improved model by AHP

Analytic hierarchy process (AHP) is a systematic method to calculate single hierarchy (weight) and total hierarchy (total weight) by qualitative index fuzzy quantization method and take it as the objective (multiple indexes) and multi-scheme optimization decision. Our goal is to use AHP to rank the weights of the three sub-models mentioned above.

The matrix is:

$$\begin{pmatrix} 1 & 3 & \frac{1}{4} \\ \frac{1}{3} & 1 & \frac{1}{6} \\ 4 & 6 & 1 \end{pmatrix}$$

Table 1. AHP production

Feature	Weight
$E_{total}{}'$	0.2176
Risk′	0.0914
Fight value′.	0.6910

Then we have final formula：

$$\text{Final } value = 0.2176E_{total}{}' + 0.0914Risk' + 0.6910Fight\ value'$$

$$s.t.\begin{cases} T_b < 43℃ \\ T_a < 60℃ \\ n < \frac{E_s - E_b - E_{fly}}{\pi(kr)^2 h\rho \frac{1}{M_{mol}} \Delta H} \end{cases} \tag{13}$$

Through this formula, we can quantify the value of the dragon, and then calculate the specific quality of the dragon and then calculate the value brought by the dragon according to this formula. And the energy consumption formula in 5.1 also helps us to solve the problem of the influence of climate factors on energy consumption.

3. Survival Area Model

In this part, we obtain the relation between the survival area required by the dragon and the quality of the dragon through the analysis of the growth curve of the dragon and the approximate replacement of the small with the large by the population density. This part is mainly composed of two models. The first model is the mass growth curve of the dragon over time, while the second model combines the mass of the dragon with the required space and pushes forward layer by layer.

3.1 Mass Time Function

Since the growth curves and sizes of the three dragons are difficult to measure, we made an assumption here that we divided each dragon into many smaller dragons, the sum of the mass of the smaller dragons equals the mass of the larger dragons, in order to make the number of dragons large enough to apply the population growth formula in biology.

Considering the dragon can grow indefinitely, we use the J type curve to simulate the growth process of the dragon, in the process of using the J type curve, we found that the growth rate of the dragon is unknown, so after considering a variety of animals decided to use the Upland Buzzard data with Logistic function fitting, due to the number of species in the K/2 when the growth rate of minimal environmental impact, so we selected the data when the K/2 as the dragon's rate of growth of approximate alternative.

$$N_{(t)} = m_s P_0^{\mathrm{t}} \tag{14}$$

$$N_{(\mathrm{t})} = \frac{K}{1+e^{-P_{\mathrm{a}}(t-t_0)}} \tag{15}$$

Where:

$N_{(t)}$ is a function of weight over time; m_s is the birth mass of the dragon
P_0 is the rate of growth; t is time
K is environmental tolerance; P_a the growth rate
t_0 is the point in time when the growth rate is fastest

Through this formula, we can get the time-dependent function of the mass of the dragon, and thus calculate the mass of the dragon.

3.2 The calculation Of Survival Area

In order to figure out the size of the survival area required by dragons, we first used the model in 6.1 to figure out the mass of dragons, and then, based on the similar method in the previous article, made it large and small, and finally used the population density to figure out the survival area required by dragons of different masses.

$$S = \beta \frac{\mathrm{m}}{\overline{m_c}} \tag{16}$$

Where:

S is the size of the field the dragon needs to survive; m is the mass of the dragon
$\overline{m}_c$ is the average mass of the bird used for the analogy; β is the population density

With this formula we can figure out the survival area of dragons of different masses.

4. Model Analysis

4.1 Sensitivity Analysis

In order to solve the problem of the impact of climate on the energy consumption of dragons, considering the impact of different temperatures and extreme cold weather on the energy consumption of birds, we used the least square method for fitting and drew the change curve of the energy consumption of dragons with body weight at -10°C to 20°C.

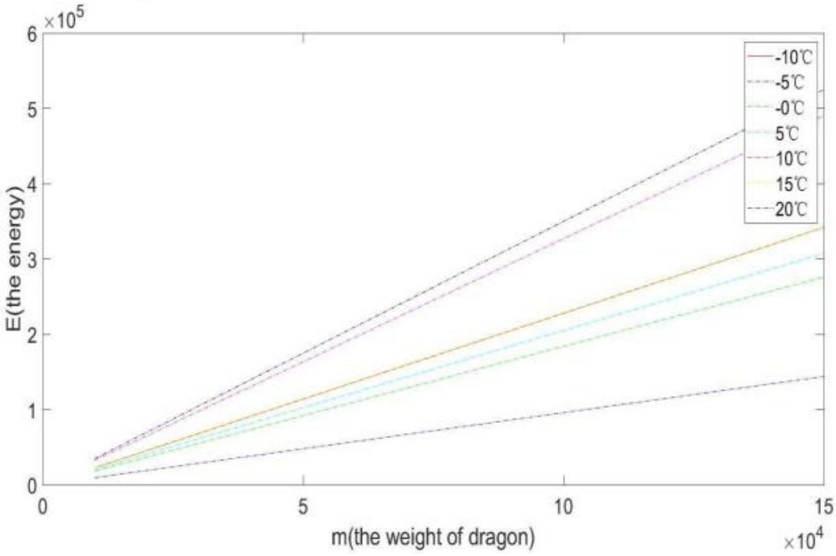

Figure 2. The function of the mass of dragon and energy with different temperature.

As shown in Figure2, when the temperature changes, the curve of the dragon's energy consumption varies greatly with the mass change, indicating that the temperature will have a relatively obvious influence on the heat consumption, which proves that this model is sensitive to temperature. From this, we can also derive the effect of climate (the only temperature is taken into account here) on the energy consumption of dragons.

4.2 The Actual Solution

In this part, we calculate the optimal mass, the minimum energy consumption and the required survival area of the dragons. In order to simplify the problem, the values we get in this part are all data of one dragon.

4.2.1 Mass Versus Living Area

By fitting the weight and time related data of Upland Buzzard, we calculated the maximum population growth rate (the rate of growth least affected by the environments) k/2=3.07056. This data was substituted into the J-shaped curve with the mass of the dragon at birth and 1 year after birth is given in the question considered. Finally, our dragon has a suitable mass of 888.93kg at the age of four.

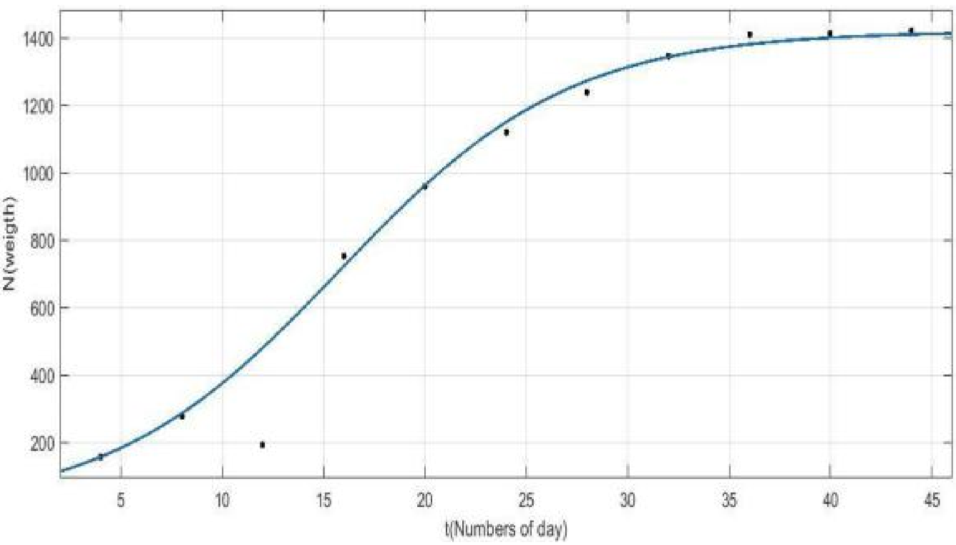

Figure 3. Curve of buteo hemiasiu's weight over time

By substituting the mass of the dragon we obtained into formula (15) and using the data we found in the BioOne Complete we figured out that the required survival area of the dragon is 226.7693 km⁄m^2.

4.2.2 Minimum energy consumption

Based on the above description, we know that the mass of the dragon is 888.93 kg. After reasonable use of the formula for calculating the energy consumption of the dragon, we find that its basic energy consumption is 113308.9 J, that is 27068.3 calories.

It should be noted that what we have calculated here is the minimum energy consumption of the dragon, that is, the energy consumption of the dragon under the condition of no fire, no flight, only basic metabolism.

5. Conclusion

We provide a detailed analysis of the conditions required to bring dragons from novels into real life and calculate the specific value of dragons. We consider the energy required for dragon survival, fire breathing and flight, and use the analytic hierarchy process to combine it with the risk of dragon death and the value of dragon according to a certain weight to form a comprehensive model. In addition, we fit the growth curve of the dragon and find out the area it needs to survive.

In the sensitivity analysis, we also studied the effects of climate on dragon energy consumption. In addition, we use relatively accurate data to give the optimal mass of the dragon, the exact value of its corresponding occupation area and energy consumption and elaborate the advantages and disadvantages of the model in detail.

Methods such as the least square method, analytic hierarchy process, and Logistic model fitting were applied in our paper.

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